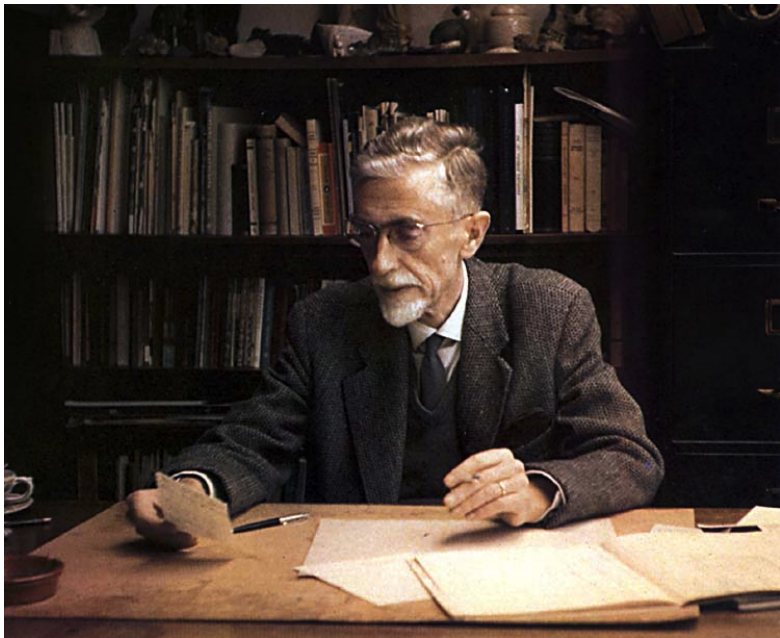


# The Mathematics of M.C. Escher

Oscar Vega

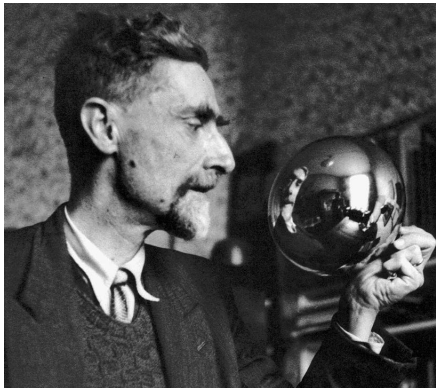
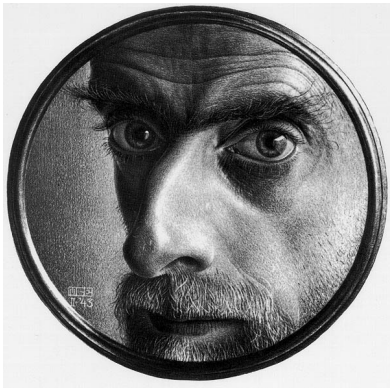
Mathematics Department Colloquium. CSU, Fresno.  
August 31, 2012



M.C. Escher in his studio (1963)

# M.C. Who?

- Maurits Cornelis Escher (1898 - 1972) was a Dutch graphic artist.
- Citizen of the world.
- Pretty unknown until 1956.



- Famous among math people.
- He had no formal mathematics training beyond secondary school.
- He did read lots of math, though.
- It all started in 1936 in Spain.

*“[The Alhambra] was the richest source of inspiration that I have ever tapped.”*



# Escher on Mathematicians

In 1957 he wrote an essay on tessellations, in which he remarked:

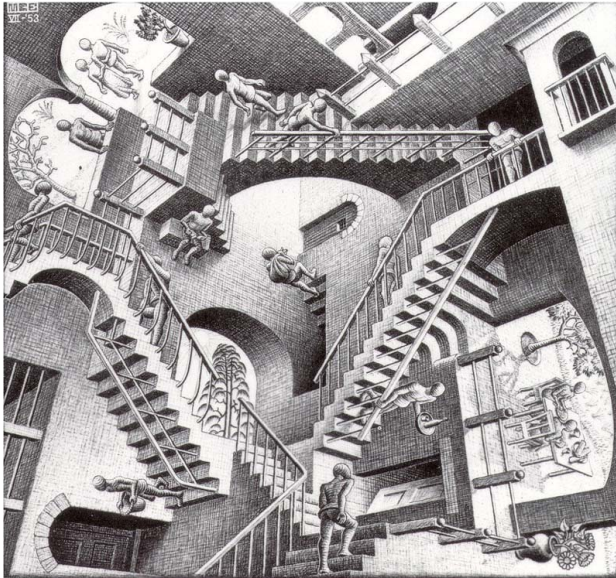
*“In mathematical quarters, the regular division of the plane has been considered theoretically... Does this mean that it is an exclusively mathematical question? In my opinion, it does not. [Mathematicians] have opened the gate leading to an extensive domain, but they have not entered this domain themselves. By their very nature they are more interested in the way in which the gate is opened than in the garden lying behind it.”*

# This Talk: Escher's Work in Three Parts

We will break Escher's work into three categories:

- Playing with  $3D$  in  $2D$ .
- Playing with different types of geometry.
- Playing with tessellations.

# Perspectives, Vanishing Points.



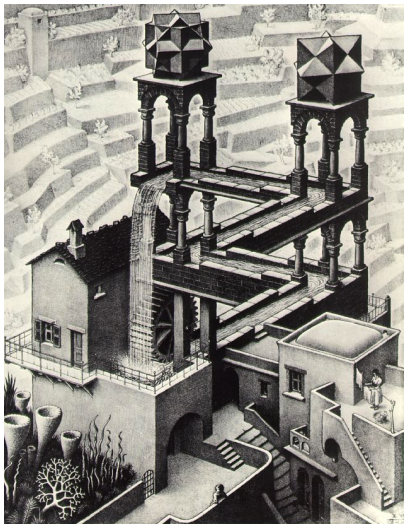
Relativity (1953. Lithograph)

# Pop Culture



Relativity (LEGO version)

# Perpetuum mobile and impossible geometry.

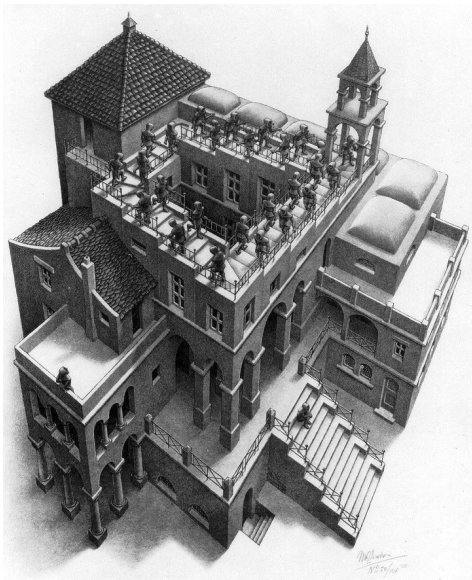


Waterfall (1961. Lithograph)



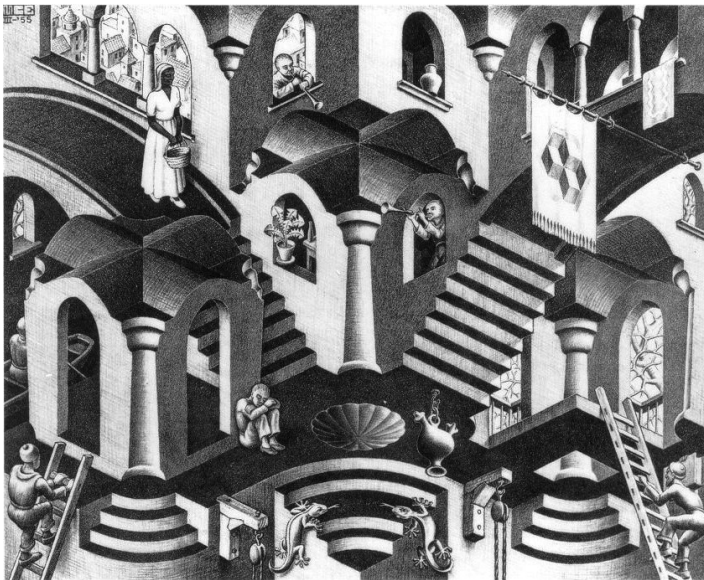
Belvedere (1958. Lithograph)

# $3D$ in $2D$



Ascending and Descending (1960. Lithograph)

# $3D$ in $2D$

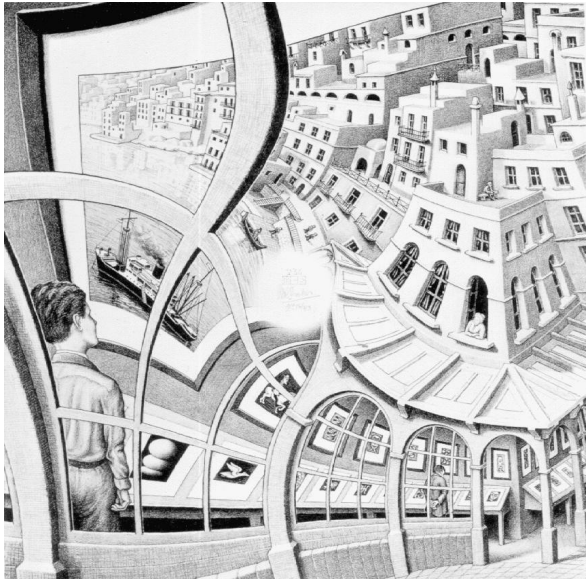


Convex and Concave (1955. Lithograph)

<http://www.youtube.com/watch?v=hAXm0dIuyug>

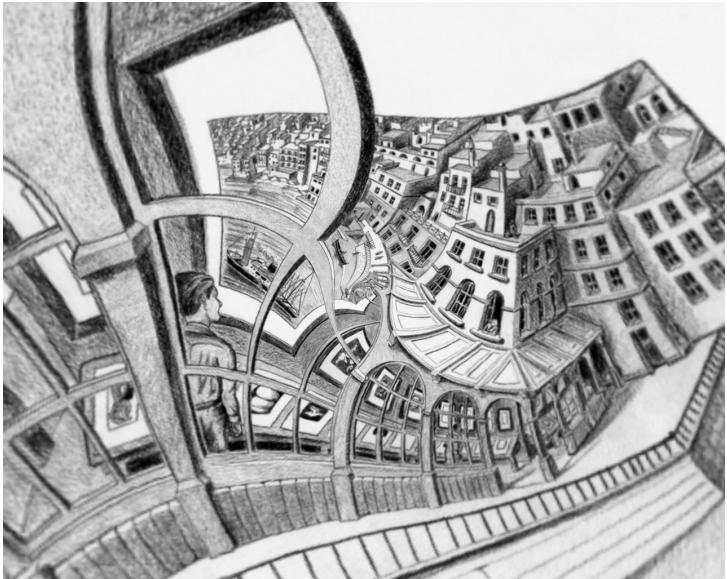


# The Droste Effect



Print Gallery (1956. Lithograph)

# The Droste Effect



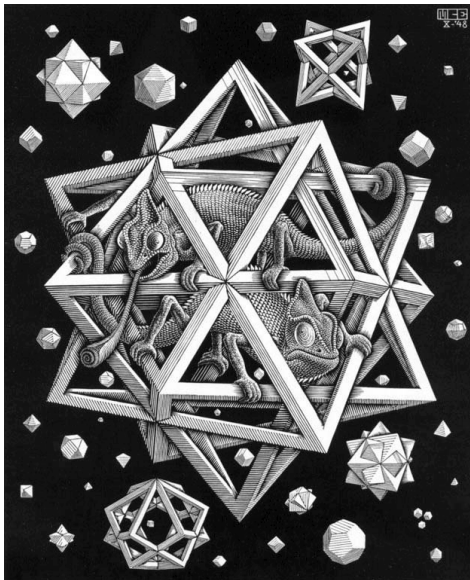
<http://www.youtube.com/watch?v=9WHdyG9mJaI>

# Pop Culture



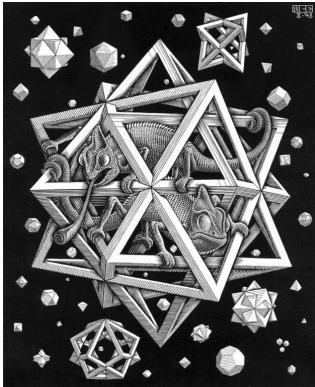


# Star Solids

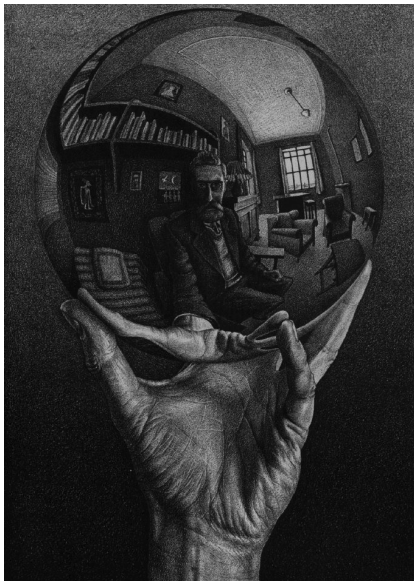


Stars (1948. Wood Engraving)

# The Compound of Three Octahedra

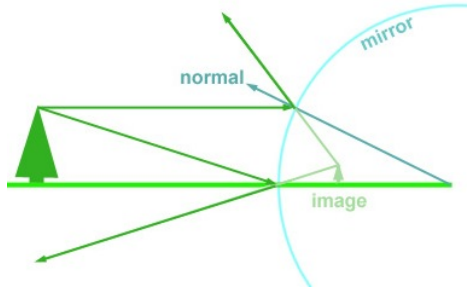
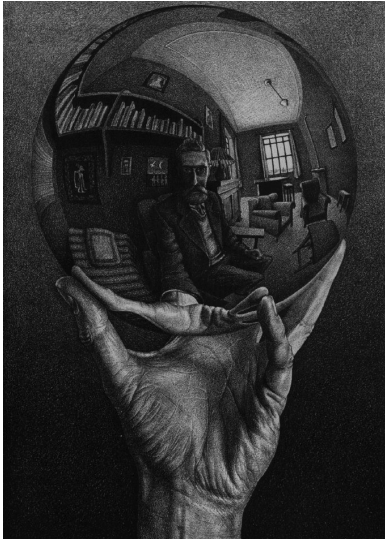


# Sphere Reflections



Hand with Reflecting Sphere (1935. Lithograph)

# Sphere Reflections

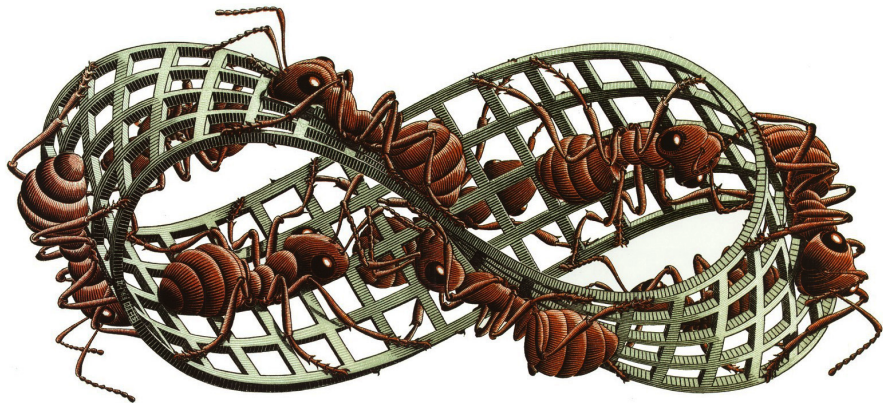




# Pop Culture

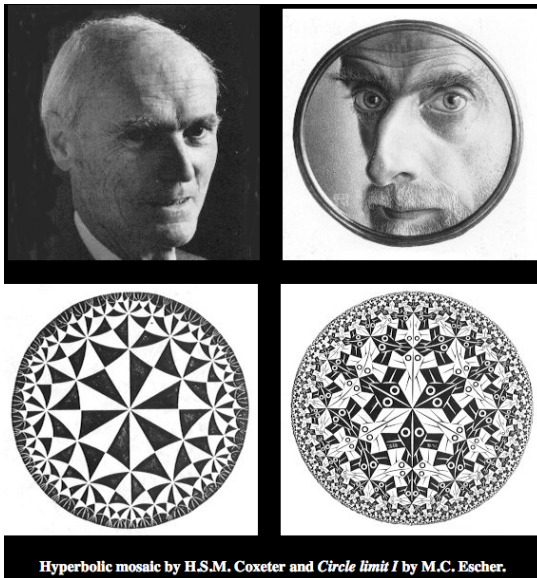


# Classic Topological Spaces



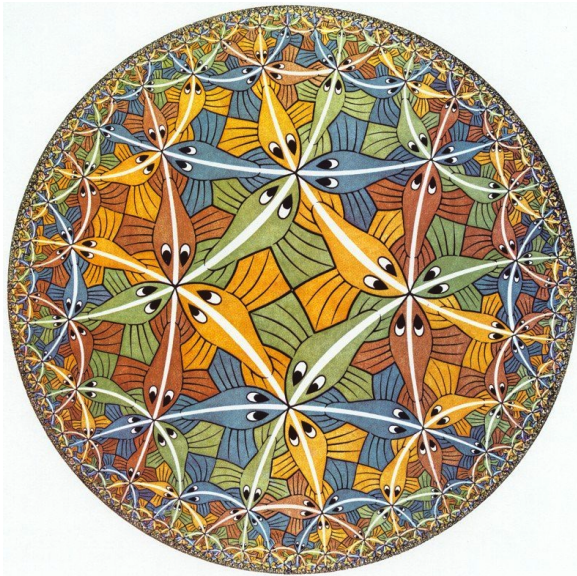
Möbius strip II (1963. Woodcut in 3 Colors)

# That Picture From Last Week's Talk



Taken from <http://www.mi.sanu.ac.rs/vismath/denes/cox.htm>

# Hyperbolic Geometry

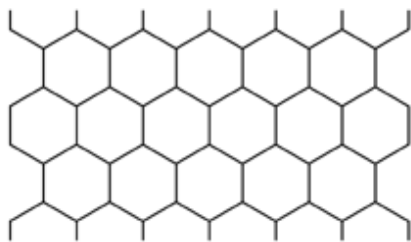
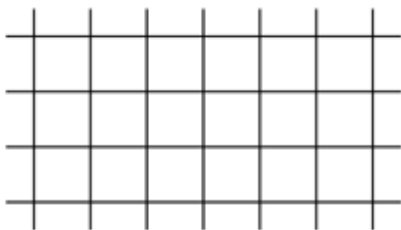
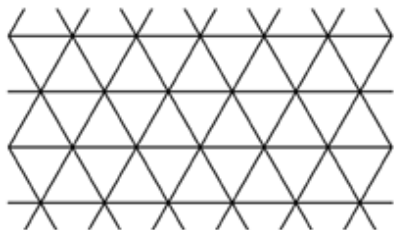


Circle Limit III (1959. Woodcut in 5 Colors)

# Tessellations

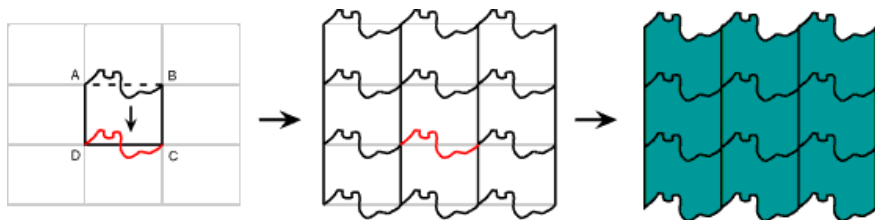
- Tessellations (or tilings) are created when a shape is repeated over and over again covering a plane without any gaps or overlaps. Some mathematicians impose two more conditions on tessellations: it must be countable and all the tiles must be closed sets.
- Of all the regular polygons, only the triangle, square, and hexagon can be used for a tessellation.
- There are many tessellations using irregular polygons.

# Regular Tessellations

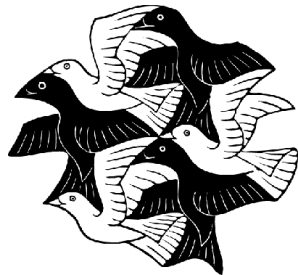
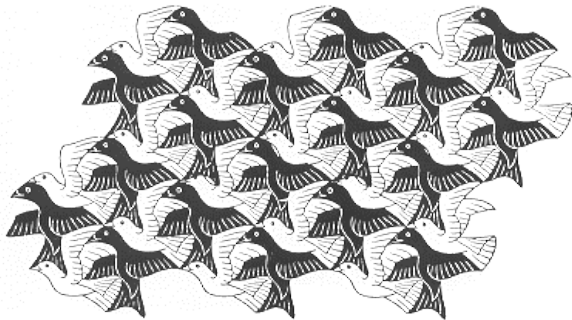


# The Trick

For tessellations that admit translations.



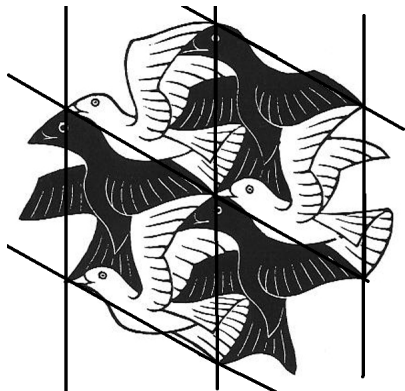
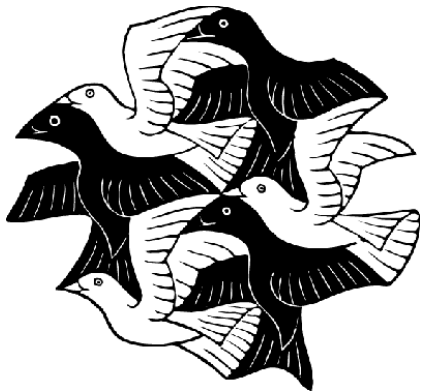
# Escher Uses The Trick



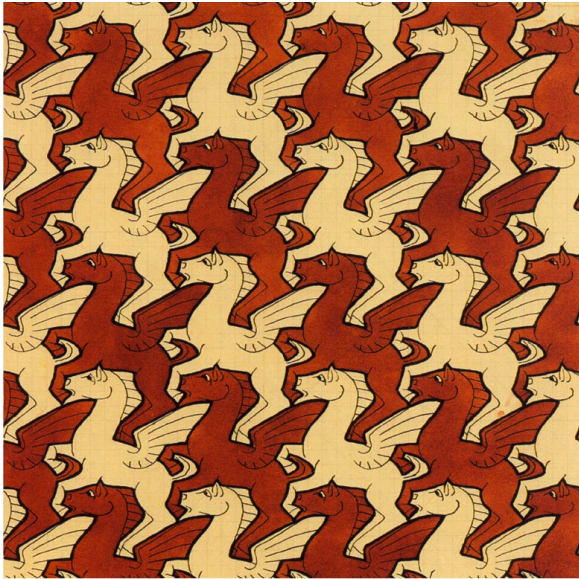
Regular Division of the Plane with Birds (1949. Wood Engraving)



# Escher Uses The Trick: Rhombi

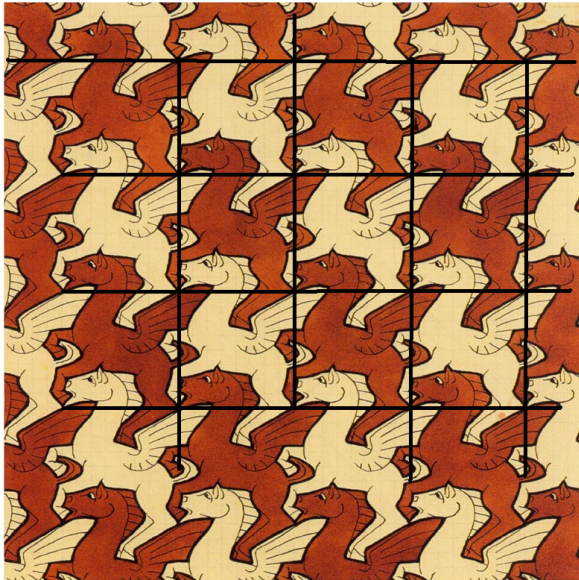


# Escher Uses The Trick

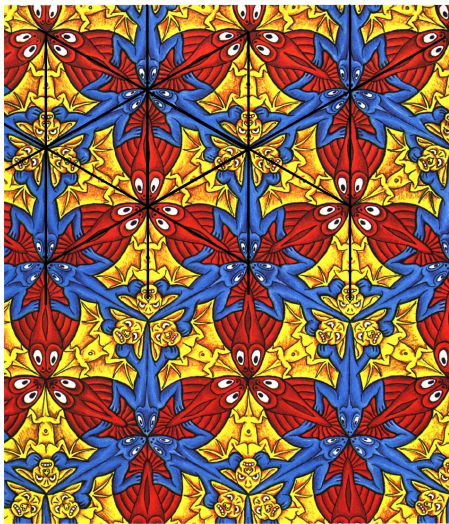


Tessellation 105 AKA Pegasus (1959. India Ink, Pencil, Watercolor)

# Escher Uses The Trick: Squares



This is Bat Country!



Lizard, fish bat (1952. Ink, Pencil, Watercolor)

# Rep-Tiles

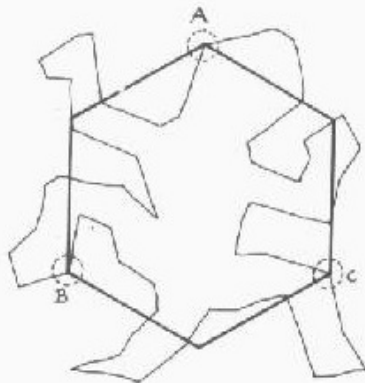
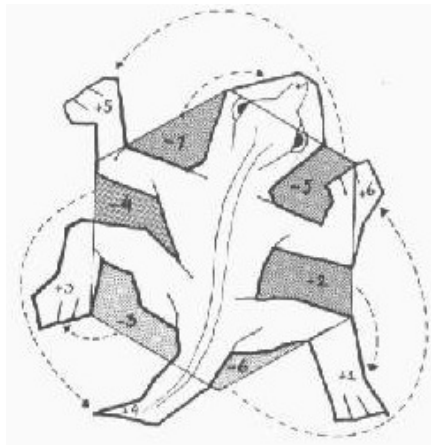


Reptiles (1943. Lithograph)

# Rep-Tiles

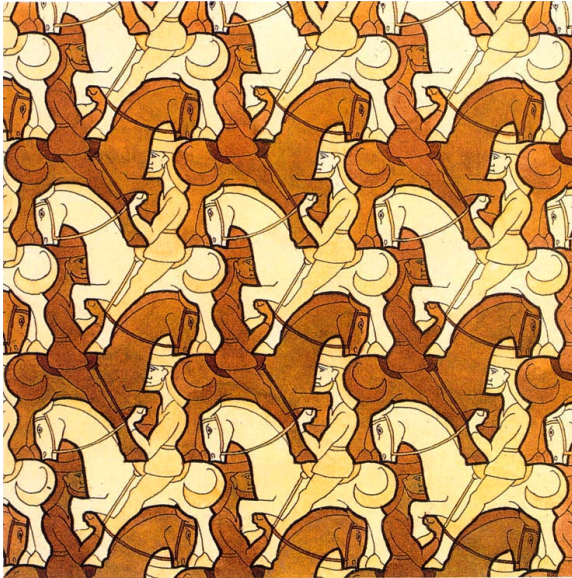


# Not as Easy as it Seems





# Tessellations Using Glide Reflections



Horseman (1946. Woodcut in 3 Colors)

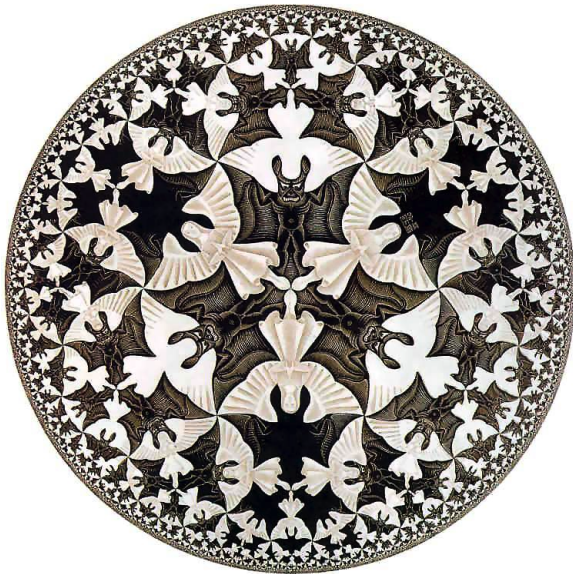


Go Bulldogs!



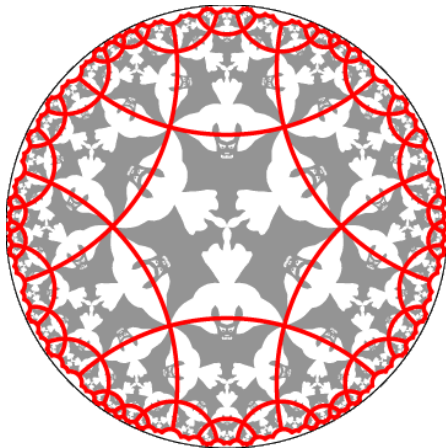
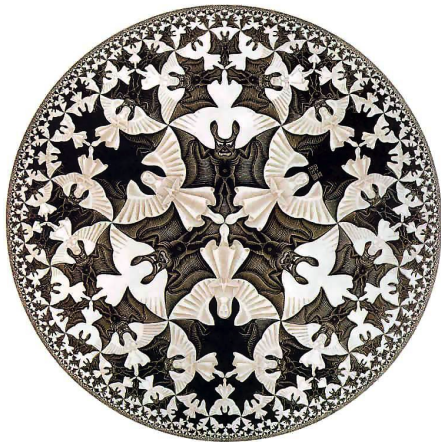
Regular Division of the Plane IV (1957. Woodcut in Red)

# Hyperbolic Tessellation?



Circle limit IV (1960. Woodcut in 2 Colors)

# $\{6, 4\}$ Hyperbolic Tessellation!



## Finally, A Quote

*“I try in my prints to testify that we live in a beautiful and orderly world, not in a chaos without norms, even though that is how it sometimes appears. My subjects are also often playful: I cannot refrain from demonstrating the nonsensicalness of some of what we take to be irrefutable certainties. It is, for example, a pleasure to deliberately mix together objects of two and three dimensions, surface and spatial relationships, and to make fun of gravity.”*

*M.C. Escher*

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