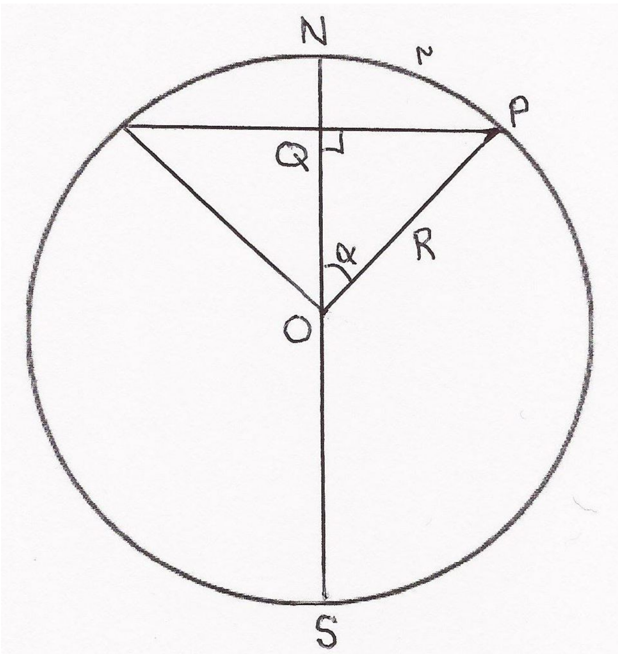
Spherical pi

[See also https://physics.illinois.edu/news/34508, but this was worked-out long before that was published.]

Like Giotto I can draw a perfect circle, but it's the rest I don't do too well. Anyway, the diagram represents a spherical Earth (with north and south indicated just as a for-instance). At the top (N) the circumference of a circular piece of real-estate is related to its diameter by the classic formula $C = \pi D$, where π (pi, approximately 3.142) is familiar to untold numbers of school-children, though probably not to estate-agents

But the larger that area of real-estate, the less accurate that formula becomes, and should it constitute the entire northern hemisphere, the effective value of pi shrinks to just 2.

I feel very uneasy at the way geometrical results about spherical objects in Euclidean three-dimensional space are supposed to apply equally to hypothetical situations in Abbott's 'Flatland' of two-dimensional space, or Riemann's positively-curved three-dimensional space, or Einstein's ditto in general relativity. But this is maths, and is unequivocal – it's true as it stands, but claims no greater generality.



R is radius of Earth (measured from the centre of the Earth at O)
Arc NP (r) is effective radius of the estate
α is the polar angle of the estate perimeter

Note that $R.\alpha = r$ if alpha is measured in radians

$QP / OP = \sin(\alpha)$, so $QP = OP \sin(\alpha) = R \sin(\alpha)$

Thus the effective diameter D of the estate is $2 \times NP = 2r$
And the effective circumference C is $\pi (2 \times QP) = 2\pi R \sin(\alpha)$

So the effective value of pi is $C/D = 2\pi R \sin(\alpha) / 2r = \pi R \sin(\alpha) / r$
which being interpreted is $\pi \sin(\alpha) / \alpha$

The factor $\sin(\alpha) / \alpha$ shrinks to unity as α diminishes.