Pythagoras revisited (Paul Glendinning p 130)

Superficially this is indeed a very slick, elegant proof – once you've sorted out the similarities, and therein lies the rub.

I have very poor powers of visualisation, and I wouldn't like to tell you how long it's taken to line up (using paper and pencil) the subsidiary triangles ΔCBD and ΔACD with the target triangle ΔABC .

First of all, however, having dropped (or floated) the perpendicular CD, it helps to label each of the subsidiary angles (1), (2), (3), (4) and find that

(1) + (2) = 90

(2) + (3) = 90

(3) + (4) = 90

(4) + (1) = 90

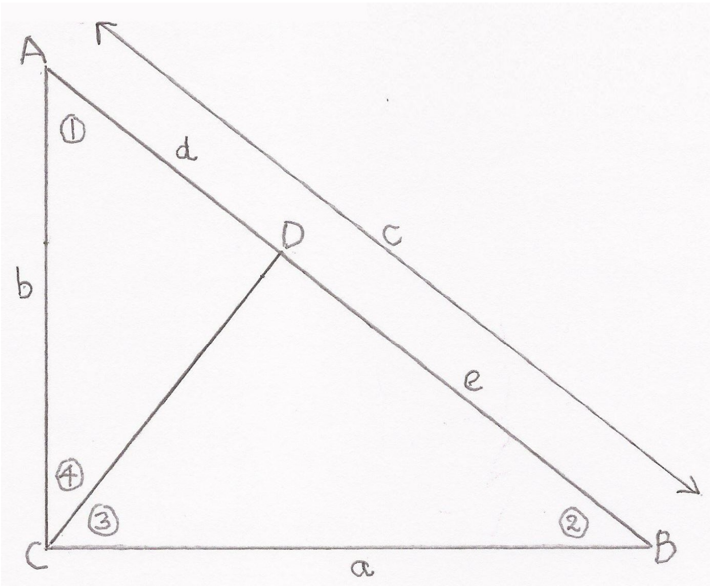
And if ABC is scalene rather than isosceles, we find by suitable subtractions that

(1) = (3)

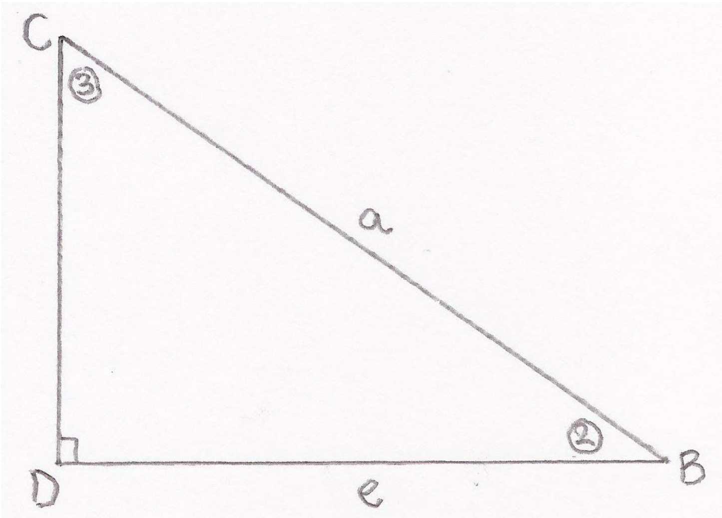
(2) = (4)

Rather by luck, (1) and (3) are slightly more than 50, while (2) and (4) are correspondingly slightly less than 40, which helps the visualisation a good deal (though please excuse the clunky diagrams).

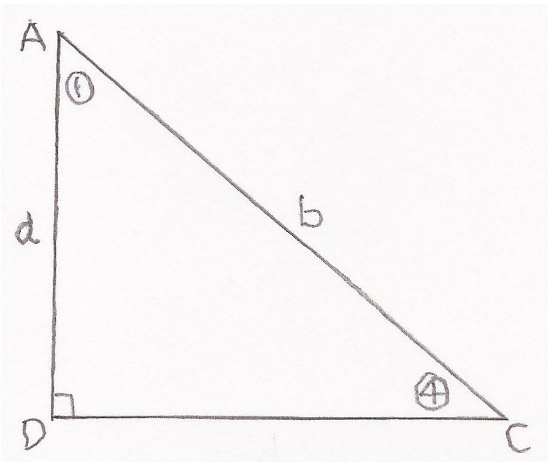
#1, ΔABC



#2, ΔCBD



#3, ΔACD



By definition, corresponding angles in similar triangles are identical, and it follows that the corresponding sides are in proportion to one another, regardless of their actual sizes. This is in fact an essential quality of Euclidean space, though there are many other equivalent criteria, which as Tom Lehrer once said, I'll tell you all about some other time.

And so we can now pick out

ΔABC ≡ ΔCBD => $c : a = a : e$ =>

=> $c/a = a/e$ => $ce = a^2$

ΔABC ≡ ΔACD => $c : b = b : d$

=> $c/b = b/d$ => $cd = b^2$

So $a^2 + b^2 = ce + cd = c(d + e) = c^2$