USE OF FOUR-FIGURE TABLES

To find the Logarithm of a given Number

The logarithm of a number consists of an integral part called the **characteristic** or **index** and a decimal part the **mantissa.**

Referring to the Tables on pages 4-5, 6-7, it will be seen that rows of four figures are placed against the numbers from 10 to 99; these four figures form the **mantissa** of a logarithm; the **index,** or **characteristic,** has to be supplied in each case.

The characteristic of any number greater than unity is positive and is less by one than the number of figures to the left of the decimal point. The characteristic of a number less than unity is negative and is greater by one than the number of zeros which follow the decimal point.

Characteristic of		6254 is 3.	Characteristic of		625400 is 5.
,,	,,	62·54 is 1.	,,	,,	6·254 is 0.
,,	,,	0·6254 is $\bar{1}$.	,,	,,	0·06254 is $\bar{2}$.
,,	,,	0·0006254 is $\bar{4}$.	,,	,,	0·00006254 is $\bar{5}$.

The latter are usually designated as *bar* 1, *bar* 2, *bar* 5, etc.

Logarithm of a number. The first two significant figures of the number are found at the extreme left of the table.

Thus, to find log 62.

In the column opposite the number 62 is found the mantissa 7924.

Hence log 62 = 1·7924.

Ex. 1. Find log 625.

Referring to the tables: find the first two digits of the number at the extreme left of the table, then passing along the horizontal line to the number in the vertical column headed by the third figure 5, we obtain the mantissa 7959.

$$\therefore \log 625 = 2{\cdot}7959.$$

The logarithm of a number consisting of four figures is found by using the mean difference columns at the extreme right.

Ex. 2. Find log 62·54.

$$\begin{aligned} \text{Mantissa of log } 625 &= \cdot 7959 \\ \text{Mean diff. for } 4 &= 3 \\ \hline \therefore \log 62{\cdot}54 &= 1{\cdot}7962 \end{aligned}$$

Similarly log 6254 = 3·7962 ; log 0·006254 = $\bar{3}$·7962.

Antilogarithms. The number corresponding to a given logarithm is obtained by using the table of antilogarithms.

Ex. 3. Find the number whose log is 1·5958.

From tables,

$$\begin{aligned} \text{Antilog } 595 &= 3936 \\ \text{Mean diff. for } 8 &= 7 \quad \text{(to be added)} \\ \hline & 3943 \end{aligned}$$

Hence the number whose log is 1·5958 is 39·43.

Similarly the number whose log is $\bar{4}$·5958 is 0·0003943.

$\bar{4}$·5958, in which the 4 only is negative, is read as bar 4, point 5, 9, 5, 8.

Multiplication and division. Multiplication of two or more numbers is effected by obtaining the sum, and division by the difference of the logarithms of the numbers. The number (obtained from the table of antilogarithms) corresponding to the sum, is the product, and the difference is the quotient.

The use of logarithms and arrangement of the work may be seen from the following examples :

Ex. 4. Multiply and divide 42·97 by 0·00258.

$$\begin{aligned} \log 42{\cdot}97 &= 1{\cdot}6332 \\ \log 0{\cdot}00258 &= \bar{3}{\cdot}4116 \\ \hline \text{Sum} &= \bar{1}{\cdot}0448 = \log 0{\cdot}1109 \\ \hline \text{Difference} &= 4{\cdot}2216 = \log 16650. \end{aligned}$$

The **sum** is obtained by noting that 1 carried from the mantissa gives +2 ; then 2 and $\bar{3} = \bar{1}$.

In subtraction, it is advisable to alter (mentally) the signs of the lower figures and add. Thus $\bar{3}$ becomes +3 and **difference** = 4·2216.

Hence 42·97 × 0·00258 = 0·1109 ; 42·97 ÷ 0·00258 = 16650.

Ex. 5. Multiply and divide 0·2543 by 0·09027.

$$\begin{aligned} \log 0{\cdot}2543 &= \bar{1}{\cdot}4053 \\ \log 0{\cdot}09027 &= \bar{2}{\cdot}9555 \\ \hline \text{Sum} &= \bar{2}{\cdot}3608 = \log 0{\cdot}02295 \\ \hline \text{Difference} &= 0{\cdot}4498 = \log 2{\cdot}817 \end{aligned}$$

The sum of the negative indices is $\bar{3}$, but 1 carried from the mantissa makes the sum to be $\bar{2}$.

∴ 0·2543 × 0·09027 = 0·02295 ; 0·2543 ÷ 0·09027 = 2·817.

Ex. 6. Compute $84{\cdot}05 \times 0{\cdot}1357 \times 1{\cdot}163$.

$$\log 84{\cdot}05 = 1{\cdot}9246$$
$$\log 0{\cdot}1357 = \bar{1}{\cdot}1325$$
$$\log 1{\cdot}163 = 0{\cdot}0656$$
$$\text{log of product} = 1{\cdot}1227$$
$$\therefore \text{ product} = 13{\cdot}26.$$

Ex. 7. Evaluate $\dfrac{9{\cdot}753 \times 10{\cdot}34 \times 0{\cdot}9252}{1{\cdot}453 \times 3{\cdot}143}$.

$$\log 9{\cdot}753 = 0{\cdot}9891$$
$$\log 10{\cdot}34 = 1{\cdot}0145$$
$$\log 0{\cdot}9252 = \bar{1}{\cdot}9662$$
$$\text{log numerator} = 1{\cdot}9698$$
$$0{\cdot}6595$$
$$\text{log result} = 1{\cdot}3103$$

$$\log 1{\cdot}453 = 0{\cdot}1623$$
$$\log 3{\cdot}142 = 0{\cdot}4972$$
$$\text{log denominator} = 0{\cdot}6595$$
$$\therefore \text{ Result} = 20{\cdot}43.$$

Involution and evolution. The square, cube, or other power of a number is found by multiplying the logarithm of the number by the index, then referring to the tables for the required value. When the logarithm of a number is partly negative and partly positive, the simplest plan to obtain a root of a number is to make the index exactly divisible by the given value of the root, and add compensating figures to the mantissa, as in the following examples :

Ex. 8. Compute (*a*) the square, (*b*) the cube, (*c*) the square root, (*d*) the cube root of 0·6254.

(*a*) $\log 0{\cdot}6254 = \bar{1}{\cdot}7962,$

$\log (0{\cdot}6254)^2 = 2 \times \bar{1}{\cdot}7962 = \bar{1}{\cdot}5924 = \log 0{\cdot}3912.$

(*b*) $\log (0{\cdot}6254)^3 = 3 \times \bar{1}{\cdot}7962 = \bar{1}{\cdot}3886 = \log 0{\cdot}2446.$

(*c*) $\log \sqrt{(0{\cdot}6254)} = \frac{1}{2}(\bar{2} + 1{\cdot}7962) = \bar{1}{\cdot}8981 = \log 0{\cdot}7909.$

(*d*) $\log \sqrt[3]{(0{\cdot}6254)} = \frac{1}{3}(\bar{3} + 2{\cdot}7962) = \bar{1}{\cdot}9321 = \log 0{\cdot}8553.$

$$\therefore (0{\cdot}6254)^2 = 0{\cdot}3912 \,;\ (0{\cdot}6254)^3 = 0{\cdot}2446 \,;\ \sqrt{0{\cdot}6254} = 0{\cdot}7909 \,;$$

$$\sqrt[3]{0{\cdot}6254} = 0{\cdot}8553.$$

In (*c*) it is necessary to divide $\bar{1}$ by 2, to keep the decimal part positive. $\bar{2} + 1$ is written for $\bar{1}$, so that the negative part can be divided exactly by 2. In (*d*) $\bar{3} + 2$ is used to replace $\bar{1}$. Both (*c*) and (*d*) should be carried out mentally.

Ex. 9. Evaluate (*a*) $(372\cdot4)^{2\cdot43}$. (*b*) $(0\cdot3724)^{2\cdot43}$. (*c*) $(0\cdot3724)^{-2\cdot43}$.

(*a*) $\log 372\cdot4 = 2\cdot5710$.

$$\log (372\cdot4)^{2\cdot43} = 2\cdot43 \times 2\cdot5710 = 6\cdot2475 = \log 1768000;$$

$$\therefore\ (372\cdot4)^{2\cdot43} = 1768000.$$

(*b*) $\log (0\cdot3724)^{2\cdot43} = 2\cdot43 \times \bar{1}\cdot5710$.

In this case a positive number is multiplied by a number partly positive and partly negative, and either of the two following methods may be used:

(i) By subtraction, $\bar{1}\cdot5710$ becomes $-0\cdot4290$.

$$-0\cdot4290 \times 2\cdot43 = -1\cdot0425 = \bar{2}\cdot9575 = \log 0\cdot09067.$$

(ii) We may multiply the two parts separately, and add.

$$\begin{aligned} 0\cdot5710 \times 2\cdot43 &= \quad 1\cdot3875 \\ -1 \times 2\cdot43 &= -2\cdot43 \\ \hline \text{log of result} &= \quad \bar{2}\cdot9575 \end{aligned}$$

$$\therefore\ \text{Result} = 0\cdot09067;$$

$$\therefore\ (0\cdot3724)^{2\cdot43} = 0\cdot09067.$$

(*c*) $\log (0\cdot3724)^{-2\cdot43} = -2\cdot43 \times \bar{1}\cdot5710$.

(i) $\therefore\ -2\cdot43 \times -0\cdot4290 = 1\cdot0425 = \log 11\cdot03.$

(ii)

$$\begin{aligned} 0\cdot5710 \times (-2\cdot43) &= -1\cdot3875 \\ (-1) \times (-2\cdot43) &= \quad 2\cdot43 \\ \hline \text{log result} &= \quad 1\cdot0425 \end{aligned}$$

$$\therefore\ \text{Result} = 11\cdot03.$$

Another method. The last number could be written in the form $\frac{1}{(0\cdot3724)^{2\cdot43}}$. In this case the logarithm of the denominator can be obtained and subtracted from log 1.

Ex. 10. If $pu^{1\cdot0646} = 479$, find the value of u when $p = 203$, the value of p when $u = 3\cdot5$.

$$u^{1\cdot0646} = \tfrac{479}{203} = 2\cdot359,$$

$$1\cdot0646 \log u = \log 2\cdot359,$$

$$\log u = \frac{\log 2\cdot359}{1\cdot0646} = 0\cdot3457;$$

$$\therefore\ u = 2\cdot217.$$

$$p = \frac{479}{(3\cdot5)^{1\cdot0646}} \text{ or } \log p = \log 479 - 1\cdot0646 \log 3\cdot5;$$

$$\therefore\ p = 126\cdot3.$$