The Monty Hall controversy

Monty Hall

A goat (one of two)

A Lincoln Continental …

… or a DeLorean

Marilyn vos Savant

In its classical form, the Monty Hall Problem (MHP) is as follows:

You are a player on a game show (*Let's Make a Deal*) and are shown three identical doors. Behind one is a car, behind the other two are goats. Monty Hall, the host of the show, asks you to choose one of the doors. You do so, but you do not open your chosen door. Monty, who knows where the car is, now opens one of the other doors. He chooses his door in accordance with the following rules:

1] Monty always opens a door that conceals a goat.
2] Monty never opens the door you initially chose.
3] If Monty can open more than one door without violating rules one and two, then he chooses his door randomly.

After Monty opens his door, he gives you the choice of sticking with your original choice or switching to the other unopened door. What should you do to maximize your chances of winning the car?

- For simplification, the prize needn't be a car, and the goats, though picturesque, are superfluous.

- The labelling of the doors is entirely arbitrary. The prize is located behind Door C just for sake of argument.

The three possible sequences of the game are as follows – there are no other outcomes to be considered. Let's look first at the objective possibilities, and then the mathematical probabilities (which is where the controversy erupted when Marilyn vos Santos commented on the expectations of the participants)

A	'Pre-select' Door A	Shown Door B is empty	Stick with Door A Switch to Door C	…Lose …Win
B				
C				
A				
B	'Pre-select' Door B	Shown Door A is empty	Stick with Door B Switch to Door C	…Lose … Win
C				
A				
B				
C	'Pre-select' Door C	Shown eg Door A is empty	Stick with Door C Switch to Door B	…Win …Lose

So switching gets the prize in two cases out of three. And sticking gets the prize in just one case out of three (equivalent to a blind guess at the outset, with no switching allowed).

More in keeping with the terminology and rules of elementary probability, we can quantify the possibilities of each step taken by contestant and host

Contestant choice	Host opens	Overall probability of getting the prize without switching	Overall probability of getting the prize by switching
	-> [0] C		
[⅓] A	or	#1 [⅓ x 1] = [⅓]	= [1] by switch to C
	-> [1] B		
	-> [0] C		
[⅓] B	or	#2 [⅓ x 1] = [⅓]	= [1] by switch to C
	-> [1] A		
	-> [½] A	#3 [⅓ x ½] = [1/6] }	
[⅓] **C**	or	} [⅓]	= [0] by switch to A or B
	-> [½] B	#4 [⅓ x ½] = [1/6] }	

Note that in every case you switch from 'Contestant choice' and 'Host opens', so

- #1 switch from A and B to C
- #2 switch from B and A to C
- #3 switch from C and A to B
- #4 switch from C and B to A

So how could there possibly be any controversy ?

Well, if the contestant is well-prepared (having read in advance an analysis of the sort outlined above) they will breeze through the procedure, switch without hesitation when “Script says switch”, accepting the overall 1/3 chance of losing ,

But if the contestant is unprepared (not having read in advance an analysis of the sort outlined above) they will breeze through the procedure, but then pause undecidedly as to how best to continue

They see a 50:50 split between their original choice and the other door that the host hasn’t opened, and so they think they will get the prize in only one case out of two, an overall 1/2 chance of losing.

The following section is an extract from

https://priceonomics.com/the-time-everyone-corrected-the-worlds-smartest/

Marilyn vos Savant’s Debacle

In September 1990, [she] devoted one of her columns in *Parade Magazine* to a reader’s question, which presented a variation of the Monty Hall Problem:

“Suppose you’re on a game show, and you’re given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what’s behind the doors, opens another door, say #3, which has a goat. He says to you, “Do you want to pick door #2?” Is it to your advantage to switch your choice of doors?”

Yes; you should switch,” she replied. “The first door has a 1/3 chance of winning, but the second door has a 2/3 chance.”

What ensued for vos Savant was a nightmarish journey, rife with name-calling, gender-based assumptions, and academic persecution

Though her answer was correct, a vast swath of academics responded with outrage. In the proceeding months, vos Savant received more than 10,000 letters — including a pair from the Deputy Director of the Center for Defense Information, and a Research Mathematical Statistician from the National Institutes of Health — all of which contended that she was entirely incompetent:

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I’ll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the odds are the same. There is enough mathematical illiteracy in this country, and we don’t need the world’s highest IQ propagating more. Shame!
Scott Smith, Ph.D.
University of Florida

May I suggest that you obtain and refer to a standard textbook on probability before you try to answer a question of this type again?
Charles Reid, Ph.D.
University of Florida

I am sure you will receive many letters on this topic from high school and college students. Perhaps you should keep a few addresses for help with future columns.
W. Robert Smith, Ph.D.
Georgia State University

You are utterly incorrect about the game show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?

E. Ray Bobo, Ph.D.
Georgetown University

You made a mistake, but look at the positive side. If all those Ph.D.'s were wrong, the country would be in some very serious trouble.
Everett Harman, Ph.D.
U.S. Army Research Institute

You are the goat!
Glenn Calkins
Western State College

Maybe women look at math problems differently than men.
Don Edwards
Sunriver, Oregon

The outcry was so tremendous that vos Savant was forced to
devote three subsequent columns to explaining why her logic was correct. Even in the wake of her well-stated, clear responses, she continued to be berated. "I still think you're wrong," wrote one man, nearly a year later. "There is such a thing as female logic."

[But] the numbers behind vos Savant's conclusion don't lie !

There are a good many other websites or YouTubes about all this, and I found

https://www.youtube.com/watch?v=4Lb-6rxZxx0

https://www.youtube.com/watch?v=7u6kFlWZOWg

https://www.youtube.com/watch?v=7WvlPgIjx_M

to be most helpful.